Application of Linear Programming to Production Systems Problem: A Critique

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Abstract

the application of linear programming to optimization problems has wider acceptance and dominance to the extent that few would question its veracity. However, many problems especially in the areas of project management appear to have defied linear programming approaches in the search for optimal solution even though many have continued to use the technique without any verification as to the authenticity of the solution. Two of such cases earlier studied exhibited this tendency – the capital rationing problem as well as that of time/ cost trade-off analysis. The problem seems to lie at those assumptions associated with model formulation, which necessitated the use of linear programming in the first instance. This paper is an attempt to look into the historical perspective with a view to eliciting more comments and researches for a more appropriate solution especially in an industrial setting.

Key words: *Linear programming, internal rate of return, capital rationing, , time/cost trade-off analysis, optimization problems*

1. Introduction

Linear programming (L.P.) is mainly concerned with a method of finding the optimum value, that of either maximizing or minimizing a function $f(x_1, x_2\ldots\ldots, x_n)$ of n real variables. In a business setting, profit maximization is always emphasized which inevitably means the minimization of some cost function. This sometimes comes in the form of primal or dual and for every primal, there must exist a corresponding dual and vice versa. This may be expressed in a simplified form as given by Akpan(1999) as:-

$a_{i1}v_1 + a_{i2}v_2 + , \ldots\ldots\ldots, a_{ik}v_k \ \{\geq , = , \leq \}b_i \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$

where i = 1, ……. , x and b is the constraint (e. g. fund availability, capacity, etc) For each of these constraints, only one sign $\geq$, = , $\leq$ would hold. The objective therefore is to seek the values of these variables, v_j satisfying equation (1) where $v_j \geq 0$; j =1-k which maximizes a linear function of the form

$Z=C_iV_i \quad ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.., C_kV_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$

Lot of materials have appeared in the open literature in the use of linear programming technique in tackling diverse problems arising from the shop floor to that of sending objects to the outer space. What seems to be in vogue is that if it did succeed in a particular area of application having some familiar characteristics, it is bound to succeed in another and effort geared towards this direction are always pursued. Two of such cases which this line of thinking have failed to provide satisfactory solutions are that of capital rationing and time/cost trade-off analysis problems is therefore the trust of this paper.

2. Capital Rationing

There are basically two modes of capital rationing, the single period and that of multi-stage as it is sometimes called. The first one relates to a situation where the capital constraint is only for one period like a year while the second involves different capital budgeting in the different years with respect to different projects are to be considered. The methods of approach are somewhat different. The first could be approached using the Excess Present Value Index (EPVI) or just Profitability Index (PI) which is a variant of Net Present Value (NPV) and a modified internal rate of return developed by Akpan (1999). It must be noted however that there was a typographical error in equation (4) of that model which rightly could have been:

$NPV = C_i /(1 + r)^i - C_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$

and simplifying further, we have:

$C_i - C_0(1 + r)^i = NPV(1 + r)^i$

$C_i = NPV(1 + r)^i + C_0(1 + r)^i$

The PI also known as cost-benefit ratio expresses the ratio of the present value of the future net cash flows over the initial cash outlay at a particular interest or discount rate. The decision rule according to some researchers (e.g. van Horne, 1980) is that as long as profitability index is equal to or greater than one that investment proposal should be accepted. He also stated that for any given project, the NPV and PI would give the same accept/reject signals. This is in line with productivity measure of output/input ratio and in both cases, the discounting/interest rate including inflation rate should remain the same.

Substantial amount of work have been done in the area of multi-stage capital rationing with the first article by Lorie and Savage (1955), and without any justification whatsoever came out with the idea of maximizing NPV and went on to formulate this as a linear programming problem. What seems to be missing in their model formulation was the discount rate used in deriving the values of the different NPVs. Weingarten (1963) used Lorie-Savage model and data to develop a comprehensive linear programming model but he too failed to make mention of the discount rate used. There has been a lot of controversy ever since Weingarten's publication, some disagreeing mostly on the discount rates (not even given in their work) which invariably affected the NPV such as Baumol and Quandt (1965), Bernhard (1969), etc. while others have suggested alternative models but still based on NPV such as Bhaskar (1976). There are lot of applications of this model in the open literature, that of Oman and Duggan (1999) is worthy of mention. Linear programming was bound to fail in the search for optimal solution because the different discount rates would produce different NPVs resulting in different solutions for the same set of investment projects. Having noticed this inconsistency, Akpan (1999) went on to use a modified internal rate of return to find a combination of projects, which would give the maximum returns on investment, hence the optimum solution.

3. Time/Cost trade-off analysis

Kelley (1961) was about the first to do an extensive work in this area in which he came out with the idea of minimizing the project overall total cost. This idea is established on a trade-off between time and cost and this model is generally referred to as time/cost trade-off analysis. He realized that there is a functional relationship between project cost and duration and that work could be speeded up by the allocation of more resources in the form of direct cost for such group of activities. Akpan (2012) stated that the main aim of time/cost trade-off was to develop a model with the overall minimum cost schedule for any given project duration bearing in mind the direct and indirect costs. Having realized that the direct cost is not necessarily linear but sometimes convex, Kelley (1961) developed a parametric linear programming flow algorithm to obtain the project cost curve. With certain assumptions and using approximate linear cost function instead of the convex, which should have been difficult to model, he suggested two pairs of time estimates for each activity, the normal and the crash with their corresponding costs. He proceeded to give a linear programming formulation in which a single objective function, that of minimizing project cost subject to the limits defined by the normal and crash points. With all these assumptions, the partial solution to the problem was obtained.

It is interesting to note that the original idea of developing a model with the overall minimum cost schedule for any given project duration bearing in mind the direct and indirect costs was jettisoned by Kelley (1961) due probably to the difficulty of incorporating the indirect cost into the model formulation similar to that of economic order quantity (EOQ) model. He even failed to provide the data for the indirect cost. In the first instance, the two cost components do not have the same characteristic, the indirect cost has a linear relationship as every increase or decrease in project duration attracts the same cost whereas the direct cost is dependent on which activity is expedited. It must be remembered that the whole concept of time/cost trade-off is built on the notion that the longer the project duration, the more one incurs in indirect cost and the lower the direct cost which is basically the cost at the normal duration. Conversely the shorter the project duration, the lower the indirect cost and the more one incurs in direct cost as a result of extra cost (crash) of those activities used for expediting exercise which may be in the form of overtime, more resources, etc. The difficulty of getting the point where the indirect cost is equal to the direct cost as in the inventory control model (that is EOQ where Ordering cost = Carrying/holding cost) necessitated the use of implicit elimination procedure (Akpan, 2001) in the search for optimal solution to this aspect of the problem. At this point, the two costs will be very close to one another with a further decrease in project duration leading to an increase in overall project cost. Many attempts have been made over the years with Levin and Kirkpatrick (1978) treating those aspects relating to the saturation point and the penalty clause, Robinson (1975) using dynamic programming model, Philips and Dessouky (1977) by means of minimal cut concept, Hegazy (1999) and Azaron (2005) using genetic algorithm in which they acknowledge the problem as combinatorial. All these authors concentrate their efforts mainly on minimal direct cost reduction without considering the indirect cost to achieve the original concept of minimizing the total project cost and probably to find the point in which it occurs.

4. Discussion

As already been acknowledged by some researchers, the two problems can be viewed as combinatorial in nature and as such NP-hard. The use of meta-heuristics such as Simulated Annealing (SA), Genetic Algorithm (GA), Artificial Neural Network (ANN), Tabu Search (TS), Fussy Logic and Control Theory have now become popular choices in the search for optimal solution for NP-hard problems. Linear programming in the two cases may therefore not be appropriate because of certain assumptions associated with model formulation, a specific discount rate for the NPV in the case of capital rationing and the required crashing point which forms a part of the constraint for the time/cost trade-off problem if one is looking for absolute minimum project total cost. Even though genetic algorithm seems to dominate the search for optimal solutions for these types of problems with a strong case made for it by Drake and Choudhry (1997) in their effort to improve upon Campbell *et al*'s model (1970) used previously by Akpan (1996) in a job-shop sequencing problem, others could equally be useful and a thorough review of these techniques are given by Garetti and Taisch (1999). Effort in this direction is ongoing and these approaches may appeal to researchers in proffering alternative solutions.

5. Conclusion

This paper is mainly to serve one purpose; a clarion call for us to seek appropriate models for tackling capital rationing problem as well as that for overall total minimum cost of a project. This has become necessary because of the inconsistency experienced in the using linear programming vis-à-vis NPV in the former and the near absence of the solution to the latter. It is also believed that there may be similar problems in other areas in which linear programming have wrongly been used in the search for optimal solution.

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